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Exchange Rates and Monetary Policy When Tradable and Nontradable Goods are Complements

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Abstract

This paper examines the implications of complementarity between tradable and nontradable goods for exchange rates and monetary policy in a two-country general equilibrium model. In doing so, it revisits well-known findings in the the New Open Economy Macroeconomics literature that exchange rates are proportional to national money supplies and that optimal monetary policies respond only to domestic shocks. These results depend on a number of simplifying assumptions, including a unitary elasticity of substitution between tradable and nontradable goods. When this assumption is replaced by a more-realistic one of complementarity, exchange rates depend on relative productivity in addition to money supplies when prices are flexible. When prices are sticky, complementarity amplifies the effect of relative money supplies on the exchange rate and creates additional spillover effects from changes of the foreign money supply on domestic consumption. With complementarity, optimal monetary policies respond to external as well as internal shocks.

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1 Introduction

This paper examines the implications of complementarity between tradable and nontradable goods for exchange rates and monetary policy. It is shown that introducing complementarity in an otherwise standard two-country general equilibrium environment significantly changes standard results regarding determinants of exchange rate movements, cross-border effects of monetary policy and optimal monetary policy.

This paper's findings include that complementarity leads to an effect of productivity shocks on exchange rates and to spillovers from productivity shocks in one sector to output and consumption in other sectors when prices are flexible. When prices are sticky, complementarity magnifies the effect of changes in relative money supplies on the exchange rate and increases the effect of changes in one country's money supply on consumption in the other country. Complementarity also means that optimal monetary policies will respond to productivity shocks in both countries.

A number of papers in the new open economy macroeconomics literature have used two-country general equilibrium models with nominal rigidities to study monetary and exchange rate policies. A seminal result established by Obstfeld and Rogoff (2000) and Corsetti and Pesenti (2001) is that monetary policy can replicate the constrained-optimal flexible price outcome in response to productivity shocks. The monetary policy which achieves this is inward-looking - i.e., responding only to domestic shocks. The expenditure-switching effect of the resulting exchange rate movement ensures that the corresponding change in relative demand for domestic and imported goods matches the flexible-price allocation.

As Canzoneri, Cumby and Diba (2005) detail, this result was derived under a set of simplifying assumptions including logarithmic utility, balanced trade and a unitary elasticity of substitution between domestic and imported goods. This paper adds to a literature that examines the implications of relaxing these assumptions. Devereux and Engel (2003) find the implications differ and that fixed exchange rates are optimal in the case when prices are set in advance in the currency of the buyer of a good ("local currency pricing"). Tille (2002) examined the case where productivity shocks occur in sectors which exist in both countries, rather than being country specific. Craighead (2012) studied the implications of factor immobility between tradable and nontradable goods sectors. Sutherland (2004) examines alternative assumptions on market structure when the elasticity of substitution between domestic and imported goods is non-unitary. Tille (2001) considered the effects of monetary shocks with a non-unitary elasticity of substitution between domestic and imported goods and Berger (2007) and Berger (2008) show that, in this case, the optimal monetary policy is no longer inward-looking.

Existing papers in this literature have either not included a nontradable goods sector or have assumed a unitary elasticity of substitution between tradable and nontradable goods. However, nontradable goods account for the majority of consumption and output and empirical evidence suggests that tradable and nontradable goods are complements - e.g., Stockman and Tesar (1995) estimate an elasticity of substitution between tradable and non-tradable goods of 0.44.

This paper allows for complementarity between tradable and nontradable goods, while maintaining the other canonical assumptions of logarithmic utility, balanced trade and a unitary elasticity between home and foreign tradable goods. When tradable and nontradable goods are complements, if prices are flexible, the exchange rate depends on relative productivity as well as relative money supplies. When prices are sticky in the producer's currency, complementarity amplifies the effect of relative money supplies on the exchange rate and creates additional spillover effects from changes of the foreign money supply on domestic consumption. Optimal monetary policies are no longer inward-looking and respond to shocks in both countries.

The model is presented below in Section 2. Section 3 derives the flexible price solution and demonstrates that complementarity creates an effect of relative productivity on exchange rates as well as spillovers from productivity in one sector to output and consumption in other sectors. The solution under nominal rigidities is presented in Section 4, which also identifies a magnification effect of changes in relative money supplies on exchange rates and additional cross-border monetary policy spillover effects due to complementarity. The welfare-based optimal monetary policy rule is found in Section 5 and it is shown to be responsive to external as well as domestic shocks.

2 Model

The model has two symmetric countries, "home" and "foreign," where foreign variables will be denoted with an asterisk. Because the countries are symmetric, the exposition will focus on home.

2.1 Production

Each country produces tradable and nontradable final goods and these goods are aggregates of differentiated varieties of intermediate goods, which are produced using a linear technology. The home-produced tradable good is denoted H, the foreign-produced tradable good is denoted F, and the home and foreign nontradable goods are denoted N and N^*, respectively. Intermediate goods varieties are indexed by $j \in [0,1]$. Output of variety j of type i is given by

$$y^i(j) = z^i L^i(j) \qquad i = H, F, N, N^* \tag{1}$$

where L is labor and z is a productivity/supply shock which is specific to sectors, but not to individual varieties. Intermediate goods varieties are assumed to be costlessly aggregated into final goods according to

$$Y^i = \left[\int_0^1 y^i(j)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \qquad i = H, F, N, N^* \tag{2}$$

where θ is the elasticity of substitution between varieties. The functional form implies the price of each type of final goods, P^i, is related to the prices of the varieties, $p^i(j)$, according to

$$P^i = \left[\int_0^1 p^i(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \qquad i = H, F, N, N^*. \tag{3}$$

Total labor used in each industry is the sum of the labor used to produce each variety, i.e.,

$$L^i = \int_0^1 L^i(j) dj \qquad i = H, F, N, N^*. \tag{4}$$

2.2 Preferences

Home and foreign representative households receive utility from consumption, C, and real money balances, $\frac{M}{P}$, and disutility from labor. The utility function of the home household is:

$$U = \ln C + \chi \ln \frac{M}{P} - \eta L^H - \eta L^N \tag{5}$$

where χ and η are weighting parameters. Consumption is a bundle of nontradable and tradable goods,

$$C = \left[(1-\gamma)^{\frac{1}{\sigma}} C^{N\frac{\sigma-1}{\sigma}} + \gamma^{\frac{1}{\sigma}} C^{T\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{6}$$

where γ is the weight on tradable goods and σ is the elasticity of substitution between tradable and nontradable goods. This is a crucial parameter of interest in this study, as complementarity is implemented by assuming $\sigma < 1$, as opposed to $\sigma = 1$ as implicitly assumed in much of the existing literature. Tradable goods consumption is a combination of foreign and home-produced tradables,

$$C^T = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} C^{F\alpha} C^{H1-\alpha} \tag{7}$$

where α is the weight on imported tradable goods. The functional form implies a standard unitary elasticity of substitution between domestic and imported tradables in order to focus on the elasticity between tradables and nontradables.

The functional forms imply the following price indexes:

$$P = \left[(1-\gamma) P^{N1-\sigma} + \gamma P^{T1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{8}$$

$$P^T = P^{F\alpha} P^{H1-\alpha} \tag{9}$$

and demand functions:

$$C^N = (1-\gamma) \left(\frac{P^N}{P} \right)^{-\sigma} C \tag{10}$$

$$C^F = \alpha\gamma \frac{P^T}{P^F} \left(\frac{P^T}{P} \right)^{-\sigma} C \tag{11}$$

$$C^H = (1-\alpha)\gamma \frac{P^T}{P^H} \left(\frac{P^T}{P} \right)^{-\sigma} C. \tag{12}$$

The home household receives wages from the labor it supplies to firms in the tradable and nontradable goods sectors, w^H and w^N, and any profits from those firms, π^H and π^N. Its initial holdings of money are M_0 and it receives a transfer of money, Tr, from the central bank, and it chooses how much money to hold and how much to consume. Its budget constraint is thus:

$$M_0 + w^H L^H + w^N L^N + \pi^H + \pi^N + Tr = PC + M. \tag{13}$$

The home household's first-order conditions imply equalization of wages between sectors between sectors and

$$w^H = w^N = w = \frac{\eta}{\chi} M \tag{14}$$

as well as

$$C = \frac{M}{\chi P}. \tag{15}$$

2.3 Market Clearing

The market clearing conditions for home- and foreign-produced tradable goods are $Y^H = C^H + C^{*H}$ and $Y^F = C^{*F} + C^F$, respectively, where C^{*H} and C^{*F} are consumption by the foreign household of home- and foreign-produced tradable goods, respectively. The market clearing conditions for nontradable goods are $Y^N = C^N$ and $Y^{N*} = C^{*N*}$ for home and foreign.

The nominal exchange rate, S, is the home currency price of foreign currency, so an increase in S represents a home currency depreciation. The law of one price holds, so the price of the foreign-produced tradable in home currency is $P^F = SP^{*F}$ and the price in foreign currency of the home-produced tradable good is $P^{*H} = \frac{1}{S} P^H$.

Because the model is static, trade will be balanced. This implies the value of home's imports and its exports (foreign's imports) are equal when expressed in the same currency, i.e.,

$$P^F C^F = S P^{*H} C^{*H}. \tag{16}$$

Substituting using the demand functions and first-order conditions from both countries gives, after some algebra,

$$S = \frac{M}{M^*} \frac{\left(\frac{P^T}{P}\right)^{1-\sigma}}{\left(\frac{P^{*T}}{P^*}\right)^{1-\sigma}}. \tag{17}$$

In the absence of complementarity (i.e., $\sigma = 1$), this reduces to the familiar condition that $S = \frac{M}{M^*}$. *With complementarity, additional adjustment of the exchange rate through its effect on the relative price of tradables is needed to maintain balanced trade.* Note that the above expression does not provide an explicit solution for S because the exchange rate enters the price indexes; $\frac{P^T}{P}$ is increasing in S and $\frac{P^{*T}}{P^*}$ is decreasing in S.

3 Flexible-Price Solution

The profits of firm j in sector i are given by:

$$\pi^i(j) = p^i(j) y^i(j) - \frac{w}{z^i} y^i(j) \tag{18}$$

$$= p^i(j) \left(\frac{p^i(j)}{P^i}\right)^{-\theta} Y^i - \frac{w}{z^i} \left(\frac{p^i(j)}{P^i}\right)^{-\theta} Y^i. \tag{19}$$

Maximizing with respect to $p^i(j)$ yields the familiar condition that prices will be set at a markup over marginal cost, $p^i(j) = \mu \frac{w}{z^i}$ where $\mu = \frac{\theta}{\theta - 1}$. Substituting the first-order condition relating the wage to the money stock gives home currency prices for nontradable and home-produced tradable goods:

$$P^N = \frac{\mu\eta}{\chi}\frac{M}{z^N} \tag{20}$$

$$P^H = \frac{\mu\eta}{\chi}\frac{M}{z^H}. \tag{21}$$

Similarly, the foreign currency price of foreign-produced tradable goods is $P^{*F} = \frac{\mu\eta}{\chi}\frac{M^*}{z^{T*}}$. Since the law of one price holds, the home currency price of foreign-produced tradables is:

$$P^F = S\frac{\mu\eta}{\chi}\frac{M^*}{z^F}. \tag{22}$$

In much of the analysis that follows, it is useful to log-linearize, where $\hat{x} \equiv \ln x - \ln \bar{x} \simeq \frac{x-\bar{x}}{\bar{x}}$ and $\bar{x}$ denotes values in the symmetric equilibrium where $z = 1$ for all sectors.

Substituting the above expressions for prices into the price indexes and then into the balanced trade condition (17) and log-linearlizing yields the following expression for the exchange rate,

$$\hat{S} = \hat{M} - \hat{M}^* + \frac{(1-\sigma)(1-\gamma)}{1-2\alpha(1-\sigma)(1-\gamma)}\left[(2\alpha-1)(\hat{z}^H - \hat{z}^F) + (\hat{z}^N - \hat{z}^{N*})\right]. \tag{23}$$

which, when $\sigma \neq 1$ includes changes in productivity in every sector as well as changes in money supplies. That is, *if prices are flexible, complementarity implies that changes in relative productivity affect exchange rates.*

Note that with $\sigma = 1$, this reduces to the familiar condition that exchange rate movements are proportional to changes in relative money stocks, $\hat{S} = \hat{M} - \hat{M}^*$. Assuming $\sigma < 1$, under most plausible parameterizations[1] the second term implies an increase relative home productivity in nontradable goods $\frac{z^N}{z^{N*}}$ leads to a home currency depreciation (i.e., $\hat{S} > 0$). Complementarity implies the

[1]I.e., where $(1-\sigma)(1-\gamma)\alpha < \frac{1}{2}$, which will be assumed for the remainder of the analysis.

increased supply of home nontradables causes an increased demand for tradables by home, including imports, so, to maintain equilibrium, the exchange rate must depreciate so that foreign tradables become relatively more expensive. The effect of a change in relative productivity in tradable goods, $\frac{z^H}{z^F}$ depends on the weight on imports, α, in the tradable bundle. If $\alpha > \frac{1}{2}$ (i.e., greater weight on imports), an increase in relative home tradable productivity depreciates the home currency because of the increase in supply of home exports relative to imports. Note that when $\alpha = \frac{1}{2}$, home and foreign tradable goods are equally weighted in both countries' consumption and thus an increase in the relative supply of one country's tradables would have the same effect in both countries, and therefore no effect on the exchange rate.

To aid interpretation, in what follows, a benchmark parameterization will be used with $\alpha = \gamma = \sigma = 0.5$, which assumes equal weight between tradables and nontradables, equal weight between domestic and imported tradables, and complementarity between tradables and nontradables. In this case (23) becomes

$$\hat{S} = \hat{M} - \hat{M}^* + \frac{1}{3}(\hat{z}^N - \hat{z}^{N*}). \tag{24}$$

where the second term illustrates how increase in home nontradable productivity leads to a depreciation because complementarity means that the increased supply of home nontradable goods leads to greater home demand for tradable goods and a larger exchange rate movement is necessary to maintain equilibrium.

The expression $\hat{S} - \hat{M} + \hat{M}^*$ isolates the additional real influences on the exchange rate due to complementarity:

$$\hat{S} - \hat{M} + \hat{M}^* = \frac{(1-\sigma)(1-\gamma)}{1 - 2\alpha(1-\sigma)(1-\gamma)} \left[(2\alpha - 1)(\hat{z}^H - \hat{z}^F) + (\hat{z}^N - \hat{z}^{N*})\right] \tag{25}$$

Consumption of each good can be found as functions of productivity, including the above expression. Under the conventionally assumed unitary elasticity of substitution, consumption and output of each good rises and falls with productivity in that sector, but is not affected by changes in productivity in other sectors. Examination of home's consumption of each good below shows that *complementarity leads to spillover effects from productivity shocks in one sector to consumption and output of other sectors.*

Consumption of home nontradable goods is:

$$\hat{C}^N = -(1-\sigma)\gamma\alpha(\hat{S} - \hat{M} + \hat{M}^*) + [(1-\sigma)(1-\gamma)+\sigma]\,\hat{z}^N \\ +(1-\sigma)\gamma\alpha\hat{z}^F + (1-\sigma)\gamma(1-\alpha)\hat{z}^H. \quad (26)$$

Note that if $\sigma = 1$, this reduces to $\hat{C}^N = \hat{z}^N$. Complementarity means that when the supply of tradables increase, from either country (i.e., positive values of $\hat{z}^F$ or $\hat{z}^H$), consumption of nontradables increases. In the benchmark case,

$$\hat{C}^N = \frac{17}{24}\hat{z}^N + \frac{1}{24}\hat{z}^{N*} + \frac{1}{8}\hat{z}^F + \frac{1}{8}\hat{z}^H. \quad (27)$$

The response to an increase nontradable sector productivity is less than one-for-one as increased supply creates increased demand for tradable goods so some labor is reallocated towards those sectors. An increase in foreign nontradable sector productivity also creates a small spillover to consumption of home nontradable goods through the $\hat{S} - \hat{M} + \hat{M}^*$ term as it leads to an a home appreciation which allows it to consume more tradables, and, with complementarity, it will also increase its nontradables consumption.

Home consumption of domestically-produced tradables is:

$$\hat{C}^H = (1-\sigma)(1-\gamma)\alpha(\hat{S} - \hat{M} + \hat{M}^*) + (1-\sigma)(1-\gamma)\hat{z}^N \\ -(1-\sigma)(1-\gamma)\alpha\hat{z}^F + [1-(1-\sigma)(1-\gamma)(1-\alpha)]\,\hat{z}^H. \quad (28)$$

When $\sigma = 1$, $\hat{C}^H = \hat{z}^H$, but when $\sigma < 1$, increases in the supply of nontradables ($\hat{z}^N$) lead to increases in home consumption of domestic tradables. Increases in the supply of foreign tradables ($\hat{z}^F$) lead to decreases in home consumption of domestic tradables as complementarity causes home to reallocate some labor to the production of nontradables.

Under the benchmark parameterization,

$$\hat{C}^H = \frac{7}{24}\hat{z}^N - \frac{1}{24}\hat{z}^{N*} - \frac{1}{8}\hat{z}^F + \frac{7}{8}\hat{z}^H \tag{29}$$

where a negative spillover from foreign nontradable productivity ($\hat{z}^{N*}$) occurs due to depreciation similarly to the effect on nontradables consumption described above.

Home consumption of imported tradables is:

$$\begin{aligned}\hat{C}^F = {} & [(1-\sigma)(1-\gamma)\alpha - 1](\hat{S} - \hat{M} + \hat{M}^*) + [1 - (1-\sigma)(1-\gamma)\alpha]\,\hat{z}^F \\ & -(1-\sigma)(1-\gamma)(1-\alpha)\hat{z}^H + (1-\sigma)(1-\gamma)\hat{z}^N \end{aligned} \tag{30}$$

which reduces to $\hat{C}^F = \hat{z}^F$ when $\sigma = 1$. For the benchmark parameterization,

$$\hat{C}^F = -\frac{1}{24}\hat{z}^N + \frac{7}{24}\hat{z}^{N*} + \frac{7}{8}\hat{z}^F - \frac{1}{8}\hat{z}^H. \tag{31}$$

The negative effect of a home tradables supply increase ($\hat{z}^H$) reflects substitution away from imported tradables. The negative effect of the increased supply of foreign nontradables ($\hat{z}^{N*}$) occurs through the appreciation in the $\hat{S} - \hat{M} + \hat{M}^*$ term that arises due to complementarity. An increase in in home nontradable productivity ($\hat{z}^N$) has two effects on $\hat{C}^F$ - a positive effect because of the increased supply of nontradables, but also a negative effect through the $\hat{S} - \hat{M} + \hat{M}^*$ term, which is slightly larger under the benchmark parameters.

Note that, since output is equal to consumption for each type of good in equilibrium, the above results also imply that output of each type of good is affected by productivity in other sectors when $\sigma \neq 1$.

4 Sticky-Price Solution

The implications of complementarity are next considered in a version of the model with nominal rigidities. Prices are assumed to be set in advance in the producer's currency, and output is demand-determined, with a bar denoting a fixed price, e.g., $\bar{P}$.

The law of one price implies the price of foreign-produced tradables in home is:

$$P^F = S\bar{P}^{*F}. \tag{32}$$

Becuase it aggregates the price of imports and domestic tradables, the exchange rate enters the home tradables price index,

$$P^T = \left(S\bar{P}^{*F}\right)^{\alpha} \left(\bar{P}^H\right)^{1-\alpha}. \tag{33}$$

Through P^T, the exchange rate also affects the overall home price index:

$$P = \left[(1-\gamma)\left(\bar{P}^N\right)^{1-\sigma} + \gamma\left(P^T\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{34}$$

In terms of log deviations from the symmetric equilibrium, the change in the home tradables price is

$$\hat{P}^T \simeq \alpha\hat{S} \tag{35}$$

and change in the overall price level is (to a first-order approximation),

$$\hat{P} \simeq \gamma\alpha\hat{S}. \tag{36}$$

Exchange rate movements determine changes in the foreign tradable and overall price indexes in a symmetric fashion, but with opposite signs - i.e., $\hat{P}^{*T} = -\alpha\hat{S}$ and $\hat{P}^{*} = -\gamma\alpha\hat{S}$.

Loglinearizing the balanced trade condition (17) and solving for the exchange rate yields

$$\hat{S} = \frac{1}{1 - 2\alpha(1-\sigma)(1-\gamma)}\left(\hat{M} - \hat{M}^{*}\right) \tag{37}$$

and letting the elasticity of the exchange rate with respect to relative money supplies be denoted ε,

$$\varepsilon = \frac{1}{1 - 2\alpha(1-\sigma)(1-\gamma)}. \tag{38}$$

Letting $\omega \equiv 2(1-\sigma)\alpha(1-\gamma)$, $\varepsilon = \frac{1}{1-\omega}$. In the conventional case of $\sigma = 1$, this reduces to the familiar result that $\varepsilon = 1$ and $\hat{S} = \hat{M} - \hat{M}^{*}$. Under the maintained assumption about plausible parameter values, $\varepsilon > 1$. This implies that *with sticky prices, complementarity between tradable and nontradable goods increases the effect of a change in relative money supplies on the exchange rate*; i.e., a ***magnification effect*** exists. With the benchmark parameters, $\varepsilon = \frac{4}{3}$.

The elasticity, ε, increases as the degree of complementarity between tradable and nontradable goods increases (i.e., as σ falls). Consider an increase in the home money supply - this would result in an increase in demand for home nontradable goods and complementarity implies that home would demand more tradable goods as well, including imported tradable goods. A larger movement in relative prices due to a change in the exchange rate would then be needed to maintain balanced trade.

The elasticity also increases with the weight on imports in tradable consumption, α, and the weight on nontradable goods in overall consumption, $1-\gamma$. As α and $1-\gamma$ increase, so does the effect of an exchange rate change on the relative price of tradables as $\hat{P}^T - \hat{P} = \alpha(1-\gamma)\hat{S}$. Recall from (17) that when $\sigma \neq 1$, additional exchange rate adjustments are needed to maintain balanced trade due to the effect of the exchange rate on the relative price of tradables.

Complementarity leads to an an increased effect of changes in the foreign money supply on overall home consumption. Recall that the household's first-order conditions imply consumption proportional to real money holdings, i.e., $C = \frac{M}{\chi P}$. Using this condition, along with the solutions for the prices in the price index, P,

$$\hat{C} = (1 - \gamma\alpha\varepsilon)\hat{M} + \gamma\alpha\varepsilon\hat{M}^* \tag{39}$$

which, when $\sigma = 1$, is $\hat{C} = (1-\gamma\alpha)\hat{M} + \gamma\alpha\hat{M}^*$; i.e., the effect of home and foreign money supplies on home consumption is proportional to the weights on domestic and imported goods in the consumption bundle. Introducing complementarity between tradables and nontradables raises the weight on foreign money.

For example, if $\sigma = 1$, while $\alpha = \gamma = 0.5$,

$$\hat{C} = \frac{3}{4}\hat{M} + \frac{1}{4}\hat{M}^*. \tag{40}$$

Reducing σ to 0.5 raises the effect of the foreign money supply, $\hat{M}^*$, on home consumption, and reduces the effect of the home money supply, $\hat{M}$:

$$\hat{C} = \frac{2}{3}\hat{M} + \frac{1}{3}\hat{M}^*. \tag{41}$$

This reflects the fact that, with $\sigma < 1$, $\varepsilon > 1$ and therefore a foreign money supply increase results in a larger home currency appreciation (and a home money supply increase results in a larger depreciation).

Recalling that consumption of each good is equal to output, examination of home's consumption of each type of good illustrates another implication of complementarity: *when $\sigma = 1$, consumption and output of each good is propotional* *to the producer's money supply; complementarity leads to spillovers with an effect of the foreign money supply on home consumption of domestically produced goods as well as an effect of the home money supply on home consumption of imported tradables.*

The demand for domestic tradable goods in home is given by

$$\hat{C}^H = [1 + (1-\sigma)(1-\gamma)\alpha\varepsilon]\,\hat{M} - (1-\sigma)(1-\gamma)\alpha\varepsilon\hat{M}^*. \tag{42}$$

When $\sigma = 1$, demand for the home tradable good rises with the home money supply, i.e., $\hat{C}^H = \hat{M}$. When $\sigma < 1$, the exchange rate enters, because it affects the price of tradable goods relative to the overall price index, $\hat{P}^T - \hat{P}$. The effect of a home monetary shock is greater because of the resulting home currency depreciation inducing substitution towards the domestic tradable good. The exchange rate effect also means that a foreign monetary expansion has a negative effect as it leads to a home appreciation and substitution in favor of the foreign tradable good. Under the benchmark parameters,

$$\hat{C}^H = \frac{7}{6}\hat{M} - \frac{1}{6}\hat{M}^*. \tag{43}$$

Similarly, complementarity creates spillovers of foreign money for consumption of imported tradable goods in home:

$$\hat{C}^F = \{[(1-\sigma)(1-\gamma)\alpha - 1]\,\varepsilon + 1\}\,\hat{M} - [(1-\sigma)(1-\gamma)\alpha - 1]\,\varepsilon\hat{M}^*. \tag{44}$$

With $\sigma = 1$, home demand for foreign tradables depends only on foreign money - i.e, $\hat{C}^F = \hat{M}^*$. Complementarity amplifies the effect of an increase in M^* while inducing a negative effect of increasing M. Under the benchmark parameters,

$$\hat{C}^F = -\frac{1}{6}\hat{M} + \frac{7}{6}\hat{M}^*. \tag{45}$$

Complementarity also introduces monetary interdependence in nontradable goods consumption. Home demand for nontradable goods is given by

$$\hat{C}^N = [1 - (1-\sigma)\gamma\alpha\varepsilon]\,\hat{M} + (1-\sigma)\gamma\alpha\varepsilon\hat{M}^*. \tag{46}$$

In the absence of complementarity, demand for nontradable goods changes with the domestic money supply, i.e., $\hat{C}^N = \hat{M}$. With complementarity, the effect of the domestic money supply is reduced, but a foreign money increase spills over positively to home nontradable goods demand. Under the benchmark parameters,

$$\hat{C}^N = \frac{5}{6}\hat{M} + \frac{1}{6}\hat{M}^*. \tag{47}$$

5 Optimal Monetary Policy

An optimal monetary policy maximizes expected welfare. As shown by Kim and Kim (2003), first-order approximations can lead to incorrect welfare implications, so a second-order approximation around the flexible-price steady state is used in this section. The predetermined prices will include risk premia.

Recall the home representative household's utility function is (5). In the flexible price steady state, labor in the home tradable and nontradable sectors are given by $\bar{L}^H = \frac{\gamma}{\eta\mu}$ and $\bar{L}^N = \frac{1-\gamma}{\eta\mu}$, respectively. Following convention, the monetary component of utility is disregarded in the welfare analysis. In the steady state, non-monetary utility is

$$\bar{U} = \ln \bar{C} - \eta\bar{L}^H - \eta\bar{L}^N \tag{48}$$

$$= \ln \bar{C} - \frac{1}{\mu}. \tag{49}$$

Following Tille (2002) and Berger (2007) the analysis focuses on the consumption component, $U^C = \ln C$. The expected utility from consumption can be approximated as:

$$EU^C \simeq \bar{U}^C + \mathrm{E}(\ln C - \ln \bar{C}) \tag{50}$$

$$EU^C \simeq \bar{U}^C - \mathrm{E}\hat{P} \tag{51}$$

where the second line uses the fact that $C = \frac{M}{\chi P}$ is exactly loglinear and $\mathrm{E}\hat{M} = 0$. Note that with the risk premia terms arising from the second-order approximation, $\hat{P}$ will be different from the first-order approximation employed in the previous sections.

Monetary policy will affect this measure of welfare through its effect on the predetermined prices of each type of good, which will affect $\mathrm{E}\hat{P}$, as shown below.

To consider how monetary policy affects the predetermined prices, consider the problem of home firm j in sector i, which sets prices to to maximize the expected utility of its owner,

$$\max_{p^i(j)} = \mathrm{E}\lambda\pi^i(j) \tag{52}$$

where λ is the lagrange multiplier on the representative household's utility-maximization problem, $\lambda = \frac{1}{PC} = \frac{\chi}{M}$. The first-order condition is

$$\mathrm{E}\lambda\left[(1-\theta)\left(\frac{p^i(j)}{P^i}\right)^{-\theta} Y^i + \theta\frac{w^H}{z^i}\left(\frac{p^i(j)}{P^i}\right)^{-\theta}\frac{1}{p^i(j)}Y^i\right] = 0. \tag{53}$$

Noting that symmetry implies $p^i(j) = P^i$, the solution for P^i is

$$P^i = \frac{\theta}{\theta-1}\frac{\mathrm{E}\lambda\frac{w}{z^i}Y^i}{\mathrm{E}\lambda Y^i}. \tag{54}$$

For the home nontradable sector, $i = N$, $Y^N = C^N = (1-\gamma)\left(\frac{P^N}{P}\right)^{-\sigma} C$. Using this, along with the conditions from the household's optimization problem, $C =$

$\frac{M}{\chi P}$ and $w = \frac{\eta}{\chi}M$, gives

$$P^N = \frac{\theta}{\theta - 1}\frac{\eta}{\chi}\frac{\mathrm{E}\frac{M}{z^N}P^{\sigma-1}}{\mathrm{E}P^{\sigma-1}}. \tag{55}$$

Approximating and taking expectations yields[2],

$$\tilde{P}^N \simeq \frac{1}{2}\mathrm{E}(\hat{M} - \hat{z}^N)^2 - (1-\sigma)\gamma\alpha\mathrm{E}(\hat{M} - \hat{z}^N)\hat{S} \tag{56}$$

where the notation $\tilde{P}$ is used to specify the predetermined prices, in terms of deviations from the steady state, taking into account the second-order terms.

The first term can be interpreted as a risk premium related to the expected volatility of demand relative to marginal cost. The second term shows that when tradable and nontradable goods are complements ($\sigma < 1$), the covariance of the exchange rate and demand relative to marginal cost has a negative effect on the price. An increase in S is a home currency depreciation which increases the home price level and decreases the demand for home nontradable goods when $\sigma < 1$. Therefore, to the extent the exchange rate covaries with marginal cost the risk to the firm is reduced because demand and marginal cost are inversely related through this channel.

For home tradable goods, after approximating and taking expectations (see appendix), the resulting expression for the the price is

$$\tilde{P}^H \simeq \frac{1}{2}\mathrm{E}(\hat{M} - \hat{z}^H)^2 + (1-\sigma)(1-\gamma)\alpha\mathrm{E}\hat{S}(\hat{M} - \hat{z}^H). \tag{57}$$

The first term represents the risk premium associated with volatility of demand relative to marginal cost and the second term reflects that an increase in S is a home currency depreciation which raises home demand for tradable goods - when this moves together with demand relative to marginal cost, it creates

[2]Details of this and all other approximations are given in the appendix.

additional risk to the producer.

Similarly, or foreign tradable goods (see appendix),

$$\tilde{P}^{*F} \simeq \frac{1}{2}(\hat{M} - \hat{z}^F)^2 - (1-\sigma)(1-\gamma)\alpha \mathrm{E}\hat{S}(\hat{M}^* - \hat{z}^F). \tag{58}$$

Note that since an increase in S is a foreign appreciation which reduces foreign demand for tradable goods, the covariance term has the opposite sign from the corresponding terms in the expression for $\tilde{P}^H$.

As shown in the appendix, expected deviation of the home price index from the nonstochastic steady state is a function of home nontradable goods prices, $\tilde{P}^N$, both countries' tradable goods prices, $\tilde{P}^{*F}$ and $\tilde{P}^H$ and expected exchange rate volatility $\mathrm{E}\hat{S}^2$

$$\mathrm{E}\hat{P} \simeq (1-\gamma)\tilde{P}^N + \gamma\alpha\tilde{P}^{*F} + \gamma(1-\alpha)\tilde{P}^H + \frac{1}{2}(1-\sigma)\gamma(1-\gamma)\alpha^2\mathrm{E}\hat{S}^2. \tag{59}$$

Given that higher prices reduce expected consumption, consistent with the consumption-based utility assumption made above in (51), optimal monetary policy for home can be thought of as minimizing $\mathrm{E}\hat{P}$. Similarly, for the foreign price index,

$$\mathrm{E}\hat{P}^* \simeq (1-\gamma)\tilde{P}^{*N} + \gamma\alpha\tilde{P}^H + \gamma(1-\alpha)\tilde{P}^{*F} + \frac{1}{2}(1-\sigma)\gamma(1-\gamma)\alpha^2\mathrm{E}\hat{S}^2. \tag{60}$$

To analyze optimal monetary policy, it is assumed that the home and foreign monetary authorities act cooperatively to maximize the sum of expected home and foreign consumption utility, i.e., $\mathrm{E}U^C + \mathrm{E}U^{C*}$. Given (51) and the analogous foreign condition, the optimal monetary policy will minimize

$$W^G = \mathrm{E}\hat{P} + \mathrm{E}\hat{P}^*. \tag{61}$$

Each country's monetary policy will respond the symmetrically to shocks in the internal nontradable, domestic tradable, imported tradable and external nontradable sectors. The home and foreign monetary rules can be represented as:

$$\hat{M} = a_I \hat{z}^N + a_D \hat{z}^H + a_M \hat{z}^F + a_E \hat{z}^{N*} \tag{62}$$

$$\hat{M}^* = a_I \hat{z}^{N*} + a_D \hat{z}^F + a_M \hat{z}^H + a_E \hat{z}^N. \tag{63}$$

Inserting the solutions for predetermined prices as well as (37) into (59) and (60) and then into (61) and substituting using the monetary rules (62) and (63), under the assumption that the variance of all four productivity shocks is σ_z^2 and that they are uncorrelated with each other, the welfare loss becomes,

$$\begin{aligned} W^G \simeq \Bigg[& (1-\gamma)\left[(a_I - 1)^2 + (a_D)^2 + (a_M)^2 + (a_E)^2\right] \\ & + \gamma\left[(a_D - 1)^2 + a_I^2 + a_M^2 + a_E^2\right] \\ & + 2(1-\gamma)(1-\sigma)\gamma\alpha\varepsilon\Big\{(a_I - a_E) + (a_M - a_D) \\ & + \alpha\varepsilon\left[(a_I - a_E)^2 + (a_M - a_D)^2\right]\Big\}\Bigg]\sigma_z^2. \end{aligned} \tag{64}$$

Note that when $\sigma = 1$, the term in braces is eliminated. Recalling that $\varepsilon = \frac{1}{1-\omega}$, the welfare loss is minimized when,

$$a_I = \frac{(1-\gamma)(1+\gamma\omega\alpha\varepsilon^2) - 0.5\gamma\omega\varepsilon}{1+2\gamma\omega\alpha\varepsilon^2} \tag{65}$$

$$a_E = \frac{(1-\gamma)\gamma\omega\alpha\varepsilon^2 + 0.5\gamma\omega\varepsilon}{1+2\gamma\omega\alpha\varepsilon^2} \tag{66}$$

$$a_D = \frac{\gamma(1+\gamma\omega\alpha\varepsilon^2) + 0.5\gamma\omega\varepsilon}{1+2\gamma\omega\alpha\varepsilon^2} \tag{67}$$

$$a_M = \frac{\gamma^2\omega\alpha\varepsilon^2 - 0.5\gamma\omega\varepsilon}{1+2\gamma\omega\alpha\varepsilon^2}. \tag{68}$$

When $\sigma = 1$, $a_I = 1 - \gamma$, $a_D = \gamma$ and $a_E = a_M = 0$, which is the familiar result that optimal monetary policies are entirely inward-looking, responding only to shocks in the domestic nontradables and domestic tradables sectors, weighted by their shares in output. This no longer holds with $\sigma \neq 1$, and *the optimal policy responds to shocks in the other country's nontradable and tradable sectors*, i.e., $a_E \neq 0$ and $a_M \neq 0$.

The responses to shocks in the domestic and external nontradable sectors sum to the share of nontradable goods in output, i.e., $a_I + a_E = 1 - \gamma$. However, compared with the $\sigma = 1$ case, optimal policy responds less to the domestic shock (i.e., $a_I < 1 - \gamma$), while responding positively to a shock in the other country's nontradable sector (i.e., $a_E > 0$). Likewise, the responses to shocks in the domestic and external tradables sectors sum to the share of tradable goods in output, i.e., $a_D + a_M = \gamma$. If the share of imported tradables in consumption is sufficiently small, $\gamma\alpha < \frac{1}{2} - \alpha(1 - \sigma)(1 - \gamma)$, the optimal policy responds negatively to shocks in the other country's tradable sector, i.e., $a_M < 0$, while responding more to domestic tradable shocks, i.e., $a_D > \gamma$. Under the benchmark parameterization, $a_I = \frac{17}{44}$, $a_E = \frac{5}{44}$, $a_D = \frac{23}{44}$ and $a_M = -\frac{1}{44}$.

6 Conclusion

This research demonstrates that complementarity between tradable and nontradable goods has significant implications for exchange rates and monetary policy in open economies. With complementarity, exchange rates depend on the relative prices of nontradable goods, as well as relative money supplies. When prices are flexible, complementarity leads to effects of relative productivity on exchange rates and spillovers of productivity shocks from one sector to consumption and output in other sectors. With nominal rigidities, complementarity magnifies the effect of changes in money supplies on the exchange rate and creates additional spillover effects of changes in the foreign money supply on domestic consumption. Complementarity means that optimal monetary policies are no-longer inward-looking and respond to shocks in the other country as well as domestic shocks.

A Appendix: Second-Order Approximations

This appendix provides some details on the derivation of the second-order approximations.

A.1 Prices

A.1.1 Home and foreign nontradable goods (P^N and P^{N*})

For the price of home nontradable goods,

$$P^N = \frac{\theta}{\theta - 1}\frac{\eta}{\chi}\frac{\mathrm{E}\frac{M}{z^N}P^{\sigma-1}}{\mathrm{E}P^{\sigma-1}}, \tag{69}$$

letting $\Omega_{N1} \equiv \mathrm{E}\frac{M}{z^N}P^{\sigma-1}$ and $\Omega_{N2} \equiv \mathrm{E}P^{\sigma-1}$,

$$\hat{P}^N \simeq \hat{\Omega}_{N1} - \hat{\Omega}_{N2} \tag{70}$$

the numerator, Ω_{N1}, can be approximated as

$$\hat{\Omega}_{N1} \simeq (\sigma - 1)\mathrm{E}\hat{P} + \frac{1}{2}\mathrm{E}\left[(\hat{M} - \hat{z}^N) + (\sigma - 1)\hat{P}\right]^2 \tag{71}$$

and the denominator is approximately,

$$\hat{\Omega}_{N2} \simeq (\sigma - 1)\mathrm{E}\hat{P} + \frac{(\sigma - 1)^2}{2}\mathrm{E}\hat{P}^2. \tag{72}$$

After expanding the squared term in $\hat{\Omega}_{N1}$ and subtracting $\hat{\Omega}_{N2}$, and using the fact that all terms in the price index, P, are predetermined except for the exchange rate so, $\hat{P} = \gamma\alpha\hat{S}$,

$$\tilde{P}^N \simeq \frac{1}{2}\mathrm{E}(\hat{M} - \hat{z}^N)^2 - (1 - \sigma)\gamma\alpha\mathrm{E}(\hat{M} - \hat{z}^N)\hat{S}. \tag{73}$$

By analogous steps, the price of foreign nontradable goods (in foreign currency) is given by

$$\tilde{P}^{N*} \simeq \frac{1}{2}\mathrm{E}(\hat{M}^* - \hat{z}^{N*})^2 + (1-\sigma)\gamma\alpha\mathrm{E}(\hat{M}^* - \hat{z}^{N*})\hat{S}^*. \tag{74}$$

A.1.2 Home tradable goods (P^H)

For home tradable goods,

$$Y^H = C^H + C^{*H} \tag{75}$$

$$= (1-\alpha)\gamma\frac{P^T}{P^H}\left(\frac{P^T}{P}\right)^{-\sigma} C + \alpha\gamma\frac{P^{T*}}{P^{*H}}\left(\frac{P^{T*}}{P^*}\right)^{-\sigma} C^*, \tag{76}$$

using $C = \frac{M}{\chi P}$, $C^* = \frac{M^*}{\chi P^*}$ and $P^{*H} = \frac{1}{S}P^H$, this becomes,

$$Y^H = (1-\alpha)\gamma\frac{P^T}{P^H}\left(\frac{P^T}{P}\right)^{-\sigma}\frac{M}{\chi P} + \alpha\gamma\frac{P^{T*}}{\frac{1}{S}P^H}\left(\frac{P^{T*}}{P^*}\right)^{-\sigma}\frac{M^*}{\chi P^*}. \tag{77}$$

Substituting for S using (17), this simplifies to

$$Y^H = \gamma\frac{1}{P^H}\left(\frac{P^T}{P}\right)^{1-\sigma}\frac{M}{\chi}. \tag{78}$$

Using $P^H(j) = P^H$, $\lambda = \frac{\chi}{M}$ and $w = \frac{\eta}{\chi}M$ the first-order condition for the firm's problem (53) yields

$$P^H = \frac{\theta}{\theta-1}\frac{\eta}{\chi}\frac{\mathrm{E}\frac{M}{Z^H}\left(\frac{P^T}{P}\right)^{1-\sigma}}{\mathrm{E}\left(\frac{P^T}{P}\right)^{1-\sigma}}. \tag{79}$$

Letting $\Omega_{H1} \equiv \left(\frac{P^T}{P}\right)^{1-\sigma}\frac{M}{z^H}$ and $\Omega_{H2} \equiv \left(\frac{P^{T*}}{P^*}\right)^{1-\sigma}$, we can write

$$\tilde{P}^H \simeq \mathrm{E}\hat{\Omega}_{H1} - \mathrm{E}\hat{\Omega}_{H2} \tag{80}$$

$$\hat{\Omega}_{H1} \simeq (1-\sigma)(\hat{P}^T - \hat{P}) + (\hat{M} - \hat{z}^H) + \frac{(1-\sigma)^2}{2}(\hat{P}^T - \hat{P})^2 + (1-\sigma)(\hat{P}^T - \hat{P})(\hat{M} - \hat{z}^H) + \frac{1}{2}(\hat{M} - \hat{z}^H)^2. \tag{81}$$

Substituting using $\hat{P}^T - \hat{P} \simeq \alpha(1-\gamma)\hat{S}$ and taking expectations gives

$$\mathrm{E}\hat{\Omega}_{H1} \simeq \frac{1}{2}\mathrm{E}(\hat{M} - \hat{z}^H)^2 + (1-\sigma)\alpha(1-\gamma)\mathrm{E}\hat{S}(\hat{M} - \hat{z}^H) + \frac{(1-\sigma)^2\alpha^2(1-\gamma)^2}{2}\mathrm{E}\hat{S}^2. \tag{82}$$

Similarly,

$$\hat{\Omega}_{H2} \simeq (1-\sigma)(\hat{P}^T - \hat{P}) + \frac{(1-\sigma)^2}{2}(\hat{P}^T - \hat{P})^2 \tag{83}$$

which yields, after substituting and taking expectations,

$$\mathrm{E}\hat{\Omega}_{H2} \simeq \frac{(1-\sigma)^2\alpha^2(1-\gamma)^2}{2}\mathrm{E}\hat{S}^2. \tag{84}$$

Thus,

$$\tilde{P}^H \simeq \frac{1}{2}\mathrm{E}(\hat{M} - \hat{z}^H)^2 + (1-\sigma)\alpha(1-\gamma)\mathrm{E}\hat{S}(\hat{M} - \hat{z}^H). \tag{85}$$

A.1.3 Foreign tradable goods (P^{*F})

The foreign-currency price of foreign tradable goods is found in a similar manner. Market clearing implies

$$Y^F = C^{*F} + C^F \tag{86}$$

$$= (1-\alpha)\gamma\frac{P^{T*}}{P^{*F}}\left(\frac{P^{T*}}{P^*}\right)^{-\sigma}C^* + \alpha\gamma\frac{P^T}{P^F}\left(\frac{P^T}{P}\right)^{-\sigma}C \tag{87}$$

$$= (1-\alpha)\gamma\frac{P^{T*}}{P^{*F}}\left(\frac{P^{T*}}{P^*}\right)^{-\sigma}\frac{M^*}{\chi P^*} + \alpha\gamma\frac{P^T}{P^F}\left(\frac{P^T}{P}\right)^{-\sigma}\frac{M}{\chi P}. \tag{88}$$

Substituting using $P^F = SP^{*F}$ and the balanced trade condition (17), this simplifies to

$$Y^F = \gamma\frac{1}{P^{*F}}\left(\frac{P^{T*}}{P^*}\right)^{1-\sigma}\frac{M^*}{\chi}. \tag{89}$$

After substituting using $P^{*F}(j) = P^{*F}$, $\lambda^* = \frac{\chi}{M^*}$ and $w^* = \frac{\eta}{\chi}$ the first-order condition for the firm's problem (53) gives,

$$P^F = \frac{\theta}{\theta - 1}\frac{\eta}{\chi}\frac{\mathrm{E}\left(\frac{P^{T*}}{P^*}\right)^{1-\sigma}\frac{M^*}{z^F}}{\mathrm{E}\left(\frac{P^{T*}}{P^*}\right)^{1-\sigma}}. \tag{90}$$

Letting $\Omega_{F1} \equiv \left(\frac{P^{T*}}{P^*}\right)^{1-\sigma}\frac{M^*}{z^F}$ and $\Omega_{F2} \equiv \gamma\left(\frac{P^{T*}}{P^*}\right)^{1-\sigma}$, we can write

$$\tilde{P}^{*F} \simeq \mathrm{E}\hat{\Omega}_{F1} - \mathrm{E}\hat{\Omega}_{F2} \tag{91}$$

where, after making the substitution $\hat{P}^{T*} - \hat{P}^* \simeq -\alpha(1-\gamma)\hat{S}$,

$$\mathrm{E}\hat{\Omega}_{F1} \simeq \frac{1}{2}(\hat{M}^* - \hat{z}^F)^2 - (1-\sigma)\alpha(1-\gamma)\mathrm{E}\hat{S}(\hat{M}^* - \hat{z}^F) + \frac{(1-\sigma)^2\alpha^2(1-\gamma)^2}{2}\mathrm{E}\hat{S}^2 \tag{92}$$

and

$$\mathrm{E}\hat{\Omega}_{F2} \simeq \frac{(1-\sigma)^2\alpha^2(1-\gamma)^2}{2}\mathrm{E}\hat{S}^2. \tag{93}$$

Hence,

$$\tilde{P}^{*F} \simeq \frac{1}{2}(\hat{M} - \hat{z}^F)^2 - (1-\sigma)\alpha(1-\gamma)\mathrm{E}\hat{S}(\hat{M}^* - \hat{z}^F). \tag{94}$$

A.1.4 Home Price Index (P)

Recall that the functional forms of the consumption bundle imply a home price level of $P = \left[(1-\gamma){P^N}^{1-\sigma} + \gamma {P^T}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$, where $P^T = {P^F}^{\alpha}{P^H}^{1-\alpha}$ and the law of one price gives $P^F = SP^{*F}$. The first-order approximation for the tradable sub-index can be written $\hat{P}^T \simeq \alpha\hat{S} + \alpha\hat{P}^{*F} + (1-\alpha)\hat{P}^H$.

The equation for the price index can be rearranged as $P^{1-\sigma} = (1-\gamma){P^N}^{1-\sigma} + \gamma {P^T}^{1-\sigma}$, which yields the following second-order approximation

$$\hat{P} + \frac{1-\sigma}{2}\hat{P}^2 \simeq (1-\gamma)\left(\hat{P}^N + \frac{1-\sigma}{2}\hat{P^N}^2\right) + \gamma\left(\hat{P}^T + \frac{1-\sigma}{2}\hat{P^T}^2\right). \tag{95}$$

Making the substitution $\hat{P}^T = \alpha\hat{S} + \alpha\hat{P}^{*F} + (1-\alpha)\hat{P}^H$ and expanding the squared terms, and then using the fact that $\hat{P}^N$, $\hat{P}^{*F}$ and $\hat{P}^H$ are already second-order, so any terms involving their squares can be dropped, gives

$$\hat{P} + \frac{1-\sigma}{2}\hat{P}^2 \simeq (1-\gamma)\hat{P}^N + \gamma\alpha\hat{P}^{*F} + \gamma(1-\alpha)\hat{P}^H + \gamma\alpha\hat{S} + \frac{\gamma(1-\sigma)\alpha^2}{2}\hat{S}^2. \quad (96)$$

An expression for $\hat{P}^2$ can be found by squaring the first-order approximation (after substituting for $\hat{P}^T$),

$$\begin{aligned}\hat{P}^2 &\simeq \Big[(1-\gamma)\hat{P}^N + \gamma(\alpha\hat{S} + \alpha\hat{P}^{*F} + (1-\alpha)\hat{P}^H)\Big]^2 \\ &\simeq \gamma^2\alpha^2\hat{S}^2\end{aligned}$$

where the second line uses the fact that $\hat{P}^N$, $\hat{P}^{*F}$ and $\hat{P}^H$ are second-order. Substituting into the expression for $\hat{P} + \frac{1-\sigma}{2}\hat{P}^2$ and rearranging gives

$$\hat{P} \simeq (1-\gamma)\hat{P}^N + \gamma\alpha\hat{P}^{*F} + \gamma(1-\alpha)\hat{P}^H + \gamma\alpha\hat{S} + \frac{(1-\sigma)\alpha^2\gamma(1-\gamma)}{2}\hat{S}^2 \quad (97)$$

or, after taking expectations:

$$\mathrm{E}\hat{P} \simeq (1-\gamma)\tilde{P}^N + \gamma\alpha\tilde{P}^{*F} + \gamma(1-\alpha)\tilde{P}^H + \frac{(1-\sigma)\alpha^2\gamma(1-\gamma)}{2}\mathrm{E}\hat{S}^2. \quad (98)$$

A.1.5 Foreign Price Index (P^*)

Following similar steps as the preceding section, the foreign price index, $P^* = \Big[(1-\gamma){P^{*N}}^{1-\sigma} + \gamma{P^{*T}}^{1-\sigma}\Big]^{\frac{1}{1-\sigma}}$, where $P^{*T} = \left(\frac{1}{S}P^H\right)^\alpha \left(P^{*F}\right)^{1-\alpha}$, we have

$$\hat{P}^* \simeq (1-\gamma)\hat{P}^{*N} - \gamma\alpha\hat{S} + \gamma\alpha\hat{P}^H + \gamma(1-\alpha)\hat{P}^{*F} + \frac{1}{2}\alpha^2\gamma(1-\gamma)(1-\sigma)\hat{S}^2. \quad (99)$$

Taking expectations gives

$$\mathrm{E}\hat{P}^* \simeq (1-\gamma)\tilde{P}^{*N} + \gamma\alpha\tilde{P}^H + \gamma(1-\alpha)\tilde{P}^{*F} + \frac{1}{2}\alpha^2\gamma(1-\gamma)(1-\sigma)\mathrm{E}\hat{S}^2. \quad (100)$$

A.1.6 Welfare Loss (W^G)

The global welfare loss is:

$$W^G = \mathrm{E}\hat{P} + \mathrm{E}\hat{P}^* \quad (101)$$

$$\simeq (1-\gamma)\tilde{P}^N + (1-\gamma)\tilde{P}^{N*} + \gamma\tilde{P}^H + \gamma\tilde{P}^{*F} + (1-\sigma)\alpha^2\gamma(1-\gamma)\mathrm{E}\hat{S}^2. \quad (102)$$

Substituting the solutions for the predetermined prices gives

$$\begin{aligned} W^G \simeq\ & \frac{1-\gamma}{2}\mathrm{E}(\hat{M} - \hat{z}^N)^2 - (1-\sigma)(1-\gamma)\gamma\alpha\mathrm{E}(\hat{M} - \hat{z}^N)\hat{S} \\ & + \frac{1-\gamma}{2}\mathrm{E}(\hat{M}^* - \hat{z}^{N*})^2 + (1-\sigma)(1-\gamma)\gamma\alpha\mathrm{E}(\hat{M}^* - \hat{z}^{N*})\hat{S} \\ & + \frac{\gamma}{2}\mathrm{E}(\hat{M} - \hat{z}^H)^2 + (1-\sigma)(1-\gamma)\gamma\alpha\mathrm{E}(\hat{M} - \hat{z}^H)\hat{S} \\ & + \frac{\gamma}{2}\mathrm{E}(\hat{M}^* - \hat{z}^F)^2 - (1-\sigma)(1-\gamma)\gamma\alpha\mathrm{E}(\hat{M}^* - \hat{z}^F)\hat{S} \\ & + (1-\sigma)\alpha^2\gamma(1-\gamma)\mathrm{E}\hat{S}^2. \end{aligned} \quad (103)$$

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